It is known that the Ruu matrix is a complex conjugate symmetric matrix (the size is 2x2 or 4x4, the following takes 4x4 as an example),

$$Ruu=\left[\begin{matrix}\begin{matrix}R\_{00}&R\_{01}\\R\_{01}^{\*}&R\_{11}\end{matrix}&\begin{matrix}R\_{02}&R\_{03}\\R\_{12}&R\_{13}\end{matrix}\\\begin{matrix}R\_{02}^{\*}&R\_{12}^{\*}\\R\_{03}^{\*}&R\_{13}^{\*}\end{matrix}&\begin{matrix}R\_{22}&R\_{23}\\R\_{23}^{\*}&R\_{33}\end{matrix}\end{matrix}\right]$$

The diagonal elements（$R\_{00}、R\_{11}、R\_{22}、R\_{33}$） are real numbers, and the non-diagonal elements（$R\_{01}、R\_{02}、R\_{03}、R\_{12}、R\_{13}、R\_{23}$）are complex numbers.

The P matrix can be obtained by LDL decomposition of Ruu, as follows

$$Ruu=LDL^{H}$$

$$P=D^{-1/2}∙L^{-1}$$

Now we want to multiply the non-diagonal elements of the Ruu matrix by the real number α to get

$$Ruu1=Ruu\*\left[\begin{matrix}\begin{matrix}1&α\\α&1\end{matrix}&\begin{matrix}α&α\\α&α\end{matrix}\\\begin{matrix}α&α\\α&α\end{matrix}&\begin{matrix}1&α\\α&1\end{matrix}\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}R\_{00}&αR\_{01}\\αR\_{01}^{\*}&R\_{11}\end{matrix}&\begin{matrix}αR\_{02}&αR\_{03}\\αR\_{12}&αR\_{13}\end{matrix}\\\begin{matrix}αR\_{02}^{\*}&αR\_{12}^{\*}\\αR\_{03}^{\*}&αR\_{13}^{\*}\end{matrix}&\begin{matrix}R\_{22}&αR\_{23}\\αR\_{23}^{\*}&R\_{33}\end{matrix}\end{matrix}\right]$$

Where \* represents the Hadamard product of two matrices. The formula of P1 corresponding to Ruu1 is as follows:

$$Ruu1=L1∙D1∙L1^{H}$$

$$P1=D1^{-1/2}∙L1^{-1}$$

Question: Is it possible to directly obtain P1 (or its approximate solution) based on P and α, without calculating LDL decomposition of Ruu1?

中文

已知Ruu矩阵为复数共轭对称阵（大小为2x2或4x4，下文以4x4为例），

$$Ruu=\left[\begin{matrix}\begin{matrix}R\_{00}&R\_{01}\\R\_{01}^{\*}&R\_{11}\end{matrix}&\begin{matrix}R\_{02}&R\_{03}\\R\_{12}&R\_{13}\end{matrix}\\\begin{matrix}R\_{02}^{\*}&R\_{12}^{\*}\\R\_{03}^{\*}&R\_{13}^{\*}\end{matrix}&\begin{matrix}R\_{22}&R\_{23}\\R\_{23}^{\*}&R\_{33}\end{matrix}\end{matrix}\right]$$

其主对角线元素（$R\_{00}、R\_{11}、R\_{22}、R\_{33}$）为实数，非主对角线元素（$R\_{01}、R\_{02}、R\_{03}、R\_{12}、R\_{13}、R\_{23}$）为复数。

对Ruu做LDL分解可求得P矩阵，如下

$$Ruu=LDL^{H}$$

$$P=D^{-1/2}∙L^{-1}$$

现欲对Ruu矩阵的非主对角线元素乘以实数$α$，得到

$$Ruu1=Ruu\*\left[\begin{matrix}\begin{matrix}1&α\\α&1\end{matrix}&\begin{matrix}α&α\\α&α\end{matrix}\\\begin{matrix}α&α\\α&α\end{matrix}&\begin{matrix}1&α\\α&1\end{matrix}\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}R\_{00}&αR\_{01}\\αR\_{01}^{\*}&R\_{11}\end{matrix}&\begin{matrix}αR\_{02}&αR\_{03}\\αR\_{12}&αR\_{13}\end{matrix}\\\begin{matrix}αR\_{02}^{\*}&αR\_{12}^{\*}\\αR\_{03}^{\*}&αR\_{13}^{\*}\end{matrix}&\begin{matrix}R\_{22}&αR\_{23}\\αR\_{23}^{\*}&R\_{33}\end{matrix}\end{matrix}\right]$$

其中\*表示矩阵的Hadamard积。则Ruu1对应的P1计算公式如下：

$$Ruu1=L1∙D1∙L1^{H}$$

$$P1=D1^{-1/2}∙L1^{-1}$$

问题：不对Ruu1作LDL分解，是否能够直接基于P和$α$，经简单运算得到P1（或其近似解）？